

# Cambridge O Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 0765613034

#### **ADDITIONAL MATHEMATICS**

4037/12

Paper 1 May/June 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

#### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

#### **INFORMATION**

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 16 pages.

DC (LO) 214475 © UCLES 2021

[Turn over

#### Mathematical Formulae

#### 1. ALGEBRA

Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^{n} = a^{n} + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^{2} + \dots + \binom{n}{r}a^{n-r}b^{r} + \dots + b^{n}$$

where *n* is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ 

Arithmetic series  $u_n = a + (n-1)d$ 

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series  $u_n = ar^{n-1}$ 

$$S_n = \frac{a(1 - r^n)}{1 - r} \ (r \neq 1)$$

$$S_{\infty} = \frac{a}{1-r} \ (|r| < 1)$$

#### 2. TRIGONOMETRY

Identities

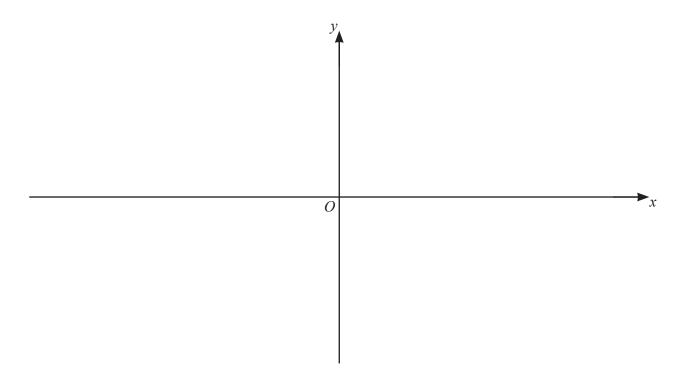
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$

Formulae for  $\triangle ABC$ 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$

1 Write  $\frac{(pqr)^{-2}r^{\frac{1}{3}}}{(p^2r)^{-1}q^3}$  in the form  $p^aq^br^c$ , where a, b and c are constants. [3]

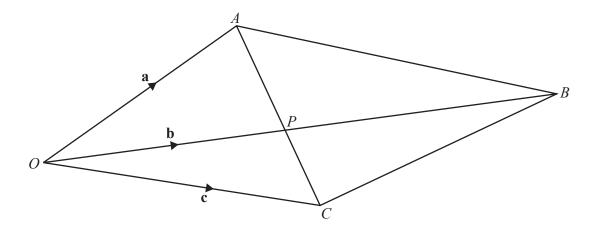
2 (a) On the axes, sketch the graph of y = |4-3x|, stating the intercepts with the coordinate axes. [2]



**(b)** Solve the inequality  $|4-3x| \ge 7$ .

[3]

3



The diagram shows the quadrilateral  $\overrightarrow{OABC}$  such that  $\overrightarrow{OA} = \mathbf{a}$ ,  $\overrightarrow{OB} = \mathbf{b}$  and  $\overrightarrow{OC} = \mathbf{c}$ . The lines  $\overrightarrow{OB}$  and  $\overrightarrow{AC}$  intersect at the point P, such that  $\overrightarrow{AP} : PC = 3 : 2$ .

(a) Find 
$$\overrightarrow{OP}$$
 in terms of a and c.

(b) Given also that 
$$OP: PB = 2:3$$
, show that  $2\mathbf{b} = 3\mathbf{c} + 2\mathbf{a}$ . [2]

A curve is such that  $\frac{d^2y}{dx^2} = (3x+2)^{-\frac{1}{3}}$ . The curve has gradient 4 at the point (2, 6.2). Find the equation of the curve.

5 (a) Given that  $\log_a p + \log_a 5 - \log_a 4 = \log_a 20$ , find the value of p. [2]

**(b)** Solve the equation 
$$3^{2x+1} + 8(3^x) - 3 = 0$$
. [3]

(c) Solve the equation  $4\log_y 2 + \log_2 y = 4$ . [3]

## 6 DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation  $y = (3 + \sqrt{5})x^2 - 8\sqrt{5}x + 60$ .

(a) Find the x-coordinate of the stationary point on the curve, giving your answer in the form  $a + b\sqrt{5}$ , where a and b are integers. [4]

(b) Hence find the y-coordinate of this stationary point, giving your answer in the form  $c\sqrt{5}$ , where c is an integer. [3]

								10	
7	(a)	A si	x-character pass	sword	l is to	be ma	de fro	om the following eight characters.	
			Digits Symbols	1	3 \$	5 #	8	9	
		No	character may b	e use	d more	e than	once	in a password.	
		Fine	d the number of	diffe	rent pa	asswor	ds tha	at can be chosen if	
		(i)	there are no res	stricti	ons,				[1]
		(ii)	the password s	tarts	with a	digit a	and fi	nishes with a digit,	[2]
	(	(iii)	the password s	tarts	with th	nree sy	/mbol	ls.	[2]
	(b)							ts selected from $n$ objects is six times $-1$ objects. Find the value of $n$ .	the number of [3]

8	Variables x and y are such that $y = Ax^b$ , where A and b are constants. When $\lg y$ is plotted against $\lg x$ , a straight line graph passing through the points (0.61, 0.57) and (5.36, 4.37) is obtained.								
	(a)	Find the value of $A$ and of $b$ .	[5]						
	Usi	ng your values of $A$ and $b$ , find							
	(b)	the value of $y$ when $x = 3$ ,	[2]						
	(c)	the value of $x$ when $y = 3$ .	[2]						

9 (a) The first three terms of an arithmetic progression are -4, 8, 20. Find the smallest number of terms for which the sum of this arithmetic progression is greater than 2000. [4]

<b>(b)</b>	The 7th and 9th terms of a	geometric progression	are 27	and 243	respectively.	Given	that 1	he
	geometric progression has a	positive common ratio,	find					

(i) this common ratio, [2]

(ii) the 30th term, giving your answer as a power of 3. [2]

(c) Explain why the geometric progression 1,  $\sin \theta$ ,  $\sin^2 \theta$ , ... for  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ , where  $\theta$  is in radians, has a sum to infinity. [2]

10 (a) Solve the equation  $\sin \alpha \csc^2 \alpha + \cos \alpha \sec^2 \alpha = 0$  for  $-\pi < \alpha < \pi$ , where  $\alpha$  is in radians. [4]

**(b)** (i) Show that 
$$\frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} = 2 \sec \theta$$
. [4]

(ii) Hence solve the equation 
$$\frac{\cos 3\phi}{1-\sin 3\phi} + \frac{1-\sin 3\phi}{\cos 3\phi} = 4$$
 for  $0^{\circ} \le \phi \le 180^{\circ}$ . [4]

### Question 11 is printed on the next page.

11 The normal to the curve  $y = \frac{\ln(x^2 + 2)}{2x - 3}$  at the point where x = 2 meets the y-axis at the point P. Find the coordinates of P. [7]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.